Motion Control of Differential Wheeled Robots with Joint Limit Constraints

J. Gonzalez-Gomez, J. G. Victores, A. Valero-Gomez, M. Abderrahim

***Abstract*— The motion of wheeled mobile robots is inherently based on their wheels' rolling capabilities. The assumption is that each wheel can rotate indefinitely, forward and backwards. This is the starting point for all motion control mechanisms of wheeled robots. In this paper, a new motion capability of differential mobile robots with limited wheel rotation capabilities is presented. The robot will be able to travel any distance and change its direction of movement even if the its wheels can not rotate within more than a certain range of angles. The proposed solution is based on the bio-inspired controller principles used for modular and legged robots, in which oscillations are generated for achieving motion. A total of two oscillators, one per wheel, are enough to generate well-coordinated rhythms on the wheels to control the robot motion. The kinematics of this new type of mobile robot motion is presented, and the relation between the oscillator's parameters and the trajectory is studied. Experiments with real robots will demonstrate the viability of this new locomotion gait.**

I. Introduction

Motion control of a mobile robot assumes that its wheels can rotate indefinitely and in any direction. Based on this principle, controllers are designed to set the speed of the wheels in order to follow a trajectory or reach a target location. This motion principle is inherent to the wheel concept, and the four basic wheel types rely on it: standard wheels, castor wheels, spherical wheels, and Swedish wheels [1]. All of these configurations are controlled by means of their wheel's angular velocities, assuming that their rotation is not constrained by any internal or external factor.

Researchers around the world are designing articulated wheeled robots [2] that include wheels and joints. The combination of wheels and joints allows robots to adapt their morphology to the terrain, increasing their maneuverability or even climbing steps [3]. These robots also have the capability of reconfiguring themselves if a wheel is broken, continuing movement (with a reduced maneuverability), thus increasing their robustness.

Fault tolerant control systems is currently a hot topic of research, including its applications in the field of mobile wheeled robots. Within this topic, there are two important research issues: first, how to detect faults from their consequences (self-aware agents) [4]; second, how to cope with faults once they are detected [5]. Among this second line of research, it is important to deal with hardware faults, that may require a reconfiguration of the robot to replace the functionality of the faulty part. Robustness can be also achieved

J. Gonzalez-gomez, J. G. Victores, A. Valero-Gomez and M. Abderrahim are members of the Robotics Lab research group within the Department of System Engineering and Automation, Universidad Carlos III de Madrid(`jggomez, jcgvicto, avgomez, mohamed)@ing.uc3m.es`

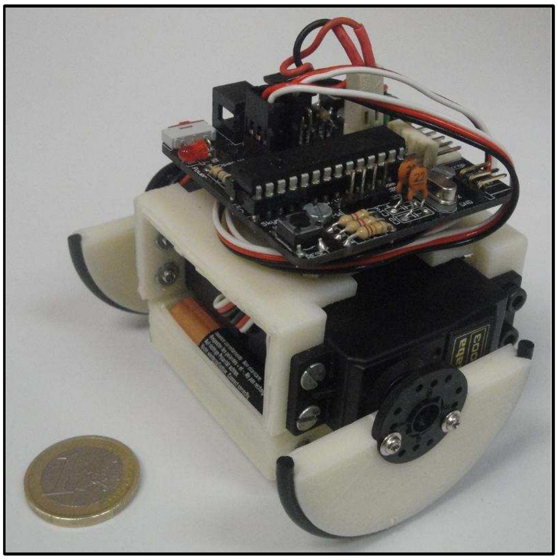

Fig. 1. A mobile robot with limited wheels, which cannot rotate 360 degrees

by means of reconfigurable robots, which are capable of adapting themselves to given tasks. In these situations robots are not dealing with faults but with unexpected situations or scenarios. Hofbaur et al. [6] have recently proposed a new wheeled modular re-configurable robot. It consists of interconnected hexagonal cells that allow the user to quickly configure/reconfigure various robot drives and change the robot's geometry. The drive units that can be attached to each module are either standard wheels or omni-directional wheels.

In 2002 Quinn et at. [7] developed the idea of *whegs*, that combines the advantages of wheels and legs. Wheels are relatively simple, and allow a vehicle to move over terrain quickly. Legs allow robots to climb obstacles that are higher than what a wheeled vehicle would be able to climb over. Whegs have also been used for climbing robots [8].

The X-RHex biologically inspired hexapedal robot [9] is the latest generation of the RHex family. It includes six legs with a half-circle shape, each one connected to a rotatory actuator. Therefore, the legs turn like standard wheels. Even if this design is mechanically simple, the robot is able to walk, run, move on rough terrains, and climb stairs.

Shen et al. [10] have designed the Quattroped, a new Leg-wheel hybrid mobile platform. The morphology of the wheels can change dynamically from a full circle into a half-circle leg, similar to the ones used by Rhex. It has great mobility on both flat grounds (by wheels) and rough terrains

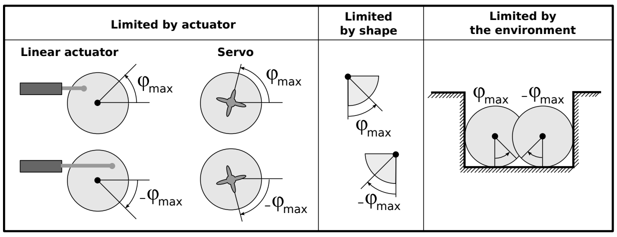

Fig. 2. The three types of limited wheels: by actuator, by shape or by the environment

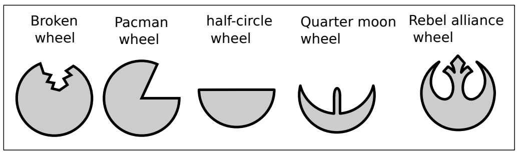

Fig. 3. Some examples of wheels limited by the shape.

(by legs).

All of these approaches rely on the "rolling principle" of the wheels, assuming that there are no limit constraints on the rotation of the wheels. This includes articulated wheeled robots, differential drive robots with fault tolerance capabilities, modular reconfigurable wheeled robots, any robot with the four basic types of wheels, and robots with whegs and Rhex-like robots among others.

The question that arises is if it is possible to provide mobility to a wheeled robot when joint limit restrictions are imposed on the wheels. If the wheels cannot rotate indefinitely, can the robot travel? How? These questions, to the best of our knowledge, have not been addressed in literature.

In this paper a new robot prototype with joint limit constraints (by design) is presented (Fig. 1). The robot's wheels can only rotate within a certain range of angles. Nonetheless, it will be proved that the robot is capable of performing reasonable 2D translations and rotations without counting on any kind of "skid" mechanism.

II. Limited wheels

A. Definition and classification

The authors define a *limited wheel* as a driven wheel that has a limitation in the rotation angle. Because of this constraint, the rotation range is less than 360 degrees: the wheel cannot turn infinitely. Limited wheels can be divided into three types (shown in figure 2) according to the nature of the constraints: limited by the actuator, limited by the shape, and limited by the environment. The first are driven by actuators that have some kinematic constraint, such as linear actuators (pneumatic cylinders, SMA-based actuators...), or servos which present mechanically built-in joint limits.

The second type comprises wheels that cannot turn 360 degrees because their shape is not a circle. This restriction is caused either by external factors, such as impacts capable of modifying the wheel's original shape, or by design criteria. A wheel limited by shape (quarter circle section) can be seen in figure 2. Its rotation range is limited to 90 degrees (from -45 to 45 degrees). A compendium of wheels limited by shape is shown in figure 3.

The third type of limitation is due to the environment constraints. Even if the wheel is not limited, objects in the environment may limit the rotation angle, acting as if the limitation was inside the wheel. An example is shown on the right of figure 3, in which the wheel is in a narrow path with walls in the front and the back. The wheel only can turn 90 degrees forward and then 90 degrees backward.

The robot shown in figure 1 is the one built for the experiments. The wheels are limited by shape (half-circle) with a rotation range of 180 degrees. The wheels' rotation is additionally limited by their actuators, as they are attached to hobby servos which also have a rotation range of 180 degrees.

B. Locomotion principle

The capacity of rolling indefinitely is inherent to the wheel concept. The authors will refer to this as the "rolling principle". On the contrary, this principle cannot be applied to limited wheels. As joint angle limits exist, the rotational angle is confined to the $\varphi \in [-\varphi_{max}, \varphi_{max}]$ restriction. Therefore, if a constant angular speed is applied to the wheel, the joint angle limits will soon be reached. Nevertheless, it is possible to apply oscillatory movements to the joint axes, with certain amplitudes and frequencies, given they do not exceed the joint limits. We will refer to this idea as the "swing principle".

Animals in nature perform rhythmic movements [11] controlled by groups of neurons called central pattern generators (CPGs). Some researchers have been applying these principles for controlling the locomotion of bio-inspired robots [12] and modular robots [13] with great success. When the CPGs reach the steady state, they behave like sinusoidal oscillators. Gonzalez-Gomez et al. [14] used these simplified oscillators to achieve the locomotion of modular snake robots. Changing the parameters of amplitude, frequency and phase difference many locomotion gaits can be performed.

In a similar way, the proposed swing principle for achieving the motion of mobile robots with limited wheels is based on sinusoidal generators. Only two of theses oscillators, one per wheel, are necessary for achieving the desired robot motion.

III. Kinematics

A. Differential drive mobile robot

The kinematic model of a differential drive robot is obtained from the equation (1), where ξ_I and ξ_R are the robot poses referred to the inertial and robot frames respectively, and $R(\theta)$ is the instant rotation matrix for transforming from the robot to the global reference frame, given by (2).

$$\dot{\xi_I} = R(\theta)\,\dot{\xi_R} \tag{1}$$

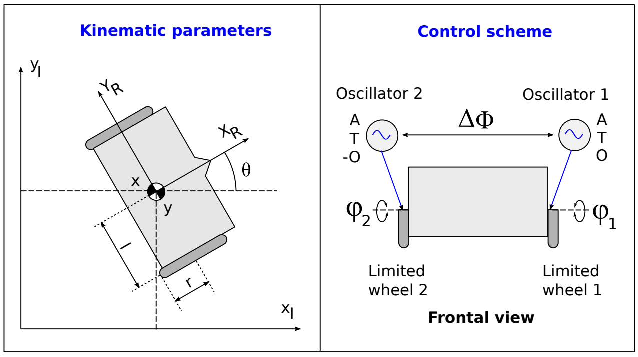

Fig. 4. On the left: Kinematic parameters and reference frames. On the right: control scheme of a differential drive robot with two limited wheels

$$R\left(\theta\right) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The robot parameters, shown in figure 4, are the wheel's radius (r), the distance from the wheel to the center of mass (l), and the wheel's rotation angles (φ_1, φ_2). As the robot is a differential drive with a castor wheel, $\dot{\xi_R}$ is given by the equation:

$$\dot{\xi_R} = \begin{bmatrix} \frac{r}{2}\left(\dot{\varphi_1} + \dot{\varphi_2}\right) \\ 0 \\ \frac{r}{2l}\left(\dot{\varphi_1} - \dot{\varphi_2}\right) \end{bmatrix} \tag{3}$$

Combining (1), (2) and (3), the equation (4) for the direct kinematics of a differential mobile robot with a castor wheel is obtained, where x and y are the robot position coordinates in the global reference frame and θ its orientation.

$$\dot{\xi_I} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\left(\dot{\varphi_1} + \dot{\varphi_2}\right)\cos\theta \\ \frac{r}{2}\left(\dot{\varphi_1} + \dot{\varphi_2}\right)\sin\theta \\ \frac{r}{2l}\left(\dot{\varphi_1} - \dot{\varphi_2}\right) \end{bmatrix} \tag{4}$$

If the initial wheel angles are set to 0, and the initial orientation is also 0, the equation (4) can be rewritten as:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \theta \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\left(\dot{\varphi_1} + \dot{\varphi_2}\right)\cos\left(\frac{r}{2l}\left(\varphi_1 - \varphi_2\right)\right) \\ \frac{r}{2}\left(\dot{\varphi_1} + \dot{\varphi_2}\right)\sin\left(\frac{r}{2l}\left(\varphi_1 - \varphi_2\right)\right) \\ \frac{r}{2l}\left(\varphi_1 - \varphi_2\right) \end{bmatrix} \tag{5}$$

which is the general kinematic model for a differential drive mobile robot with standard wheels, that moves according to the *rolling principle.*

B. Oscillating wheels

As previously stated, the locomotion of a differential drive mobile robot with limited wheels follows the *swing principle*. The wheels cannot turn indefinitely, but they can oscillate. In the model we propose, the two wheels are oscillating sinusoidally, as shown in figure 4, according to the following equations:

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} A\sin\left(\frac{2\pi}{T}t + \phi_0\right) + O \\ A\sin\left(\frac{2\pi}{T}t + \Delta\phi + \phi_0\right) - O \end{bmatrix} \tag{6}$$

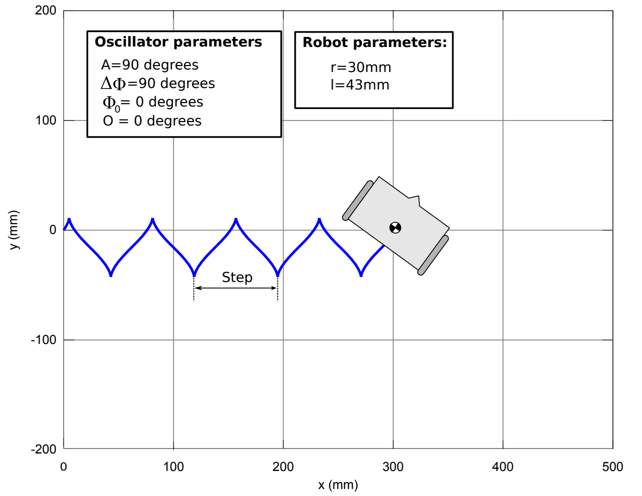

Fig. 5. Typical trajectory described by a differential mobile robot with limited wheels controlled by means of two oscillators

where the amplitude (A), period (T) and initial phase (ϕ_0) are the same for the two wheels. The offset (O) is also the same in absolute value but with different sign. There is a phase difference ($\Delta\phi$) between the right and left wheels. Combining (6) with (5), the kinematic equations of differential mobile robot with limited wheels are obtained:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \theta \end{bmatrix} = \begin{bmatrix} \frac{\pi r A}{T}\left(C\left(0\right) + C\left(\Delta\phi\right)\right)\cos\theta \\ \frac{\pi r A}{T}\left(C\left(0\right) + C\left(\Delta\phi\right)\right)\sin\theta \\ \frac{r}{2l}\left(AS\left(0\right) - AS\left(\Delta\phi\right) + 2O\right) \end{bmatrix} \tag{7}$$

where the $C(x)$ and $S(x)$ functions are defined as:

$$C\left(x\right) = \cos\left(\frac{2\pi n}{T} + \phi_0 + x\right)$$

$$S\left(x\right) = \cos\left(\frac{2\pi n}{T} + \phi_0 + x\right)$$

IV. Controlling the Movement

A. Trajectory and step

The equation (5) is used to compute the trajectory described by a differential mobile robot with limited wheels controlled by means of two sinusoidal oscillators. In figure 5 the trajectory is shown, using the same parameters than the robot built for the experiments. The robot is moving sideways along the x axis describing a periodic path. The distance traveled by the robot during one period is defined as the *step*. Following, the influence of the oscillator parameters on the movement will be explained.

The initial phase (ϕ_0) determines the initial robot pose relative to the path. It has no effect on the locomotion in the steady state. Therefore ϕ_0 is only used to calculate the initial robot orientation and linear x and y velocities ($\dot{x}$,$\dot{y}$).

The phase difference ($\Delta\phi$) is the most import parameter as it establishes the coordination between the two wheels. It has a great impact on the trajectory shape and step. If the two wheels are oscillating in phase ($\Delta\phi = 0$) the robot moves

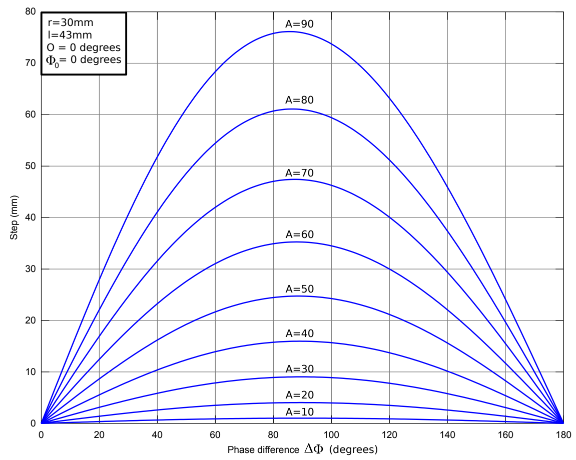

Fig. 6. Variation of the step with the phase difference for different amplitudes

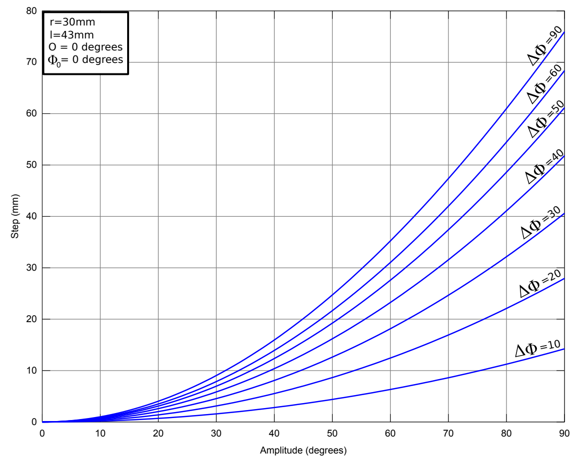

Fig. 7. Variation of the step with the amplitude for different phase differences

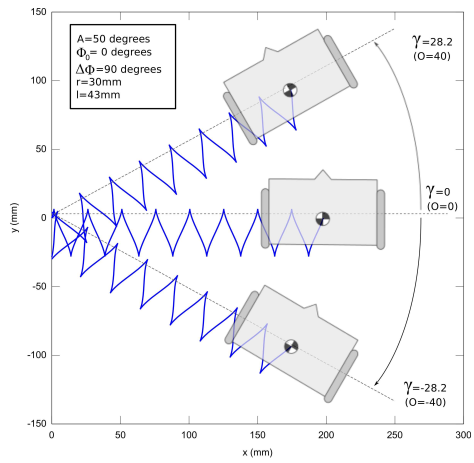

Fig. 8. Robot trajectories in three directions

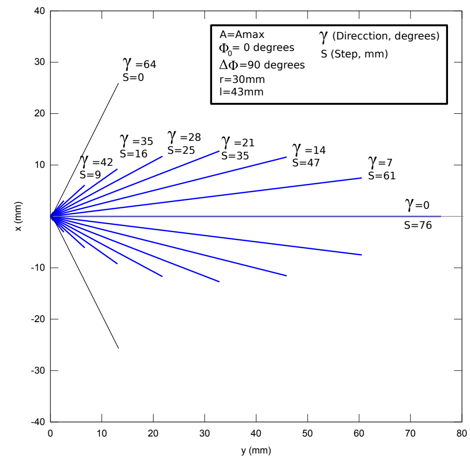

Fig. 9. Graphical representation of the step and direction using the maximum amplitude. When γ is increased, the step is smaller

forward and backward along the y axis, maintaining the same orientation ($\theta = 0$) but with no displacement on the x axis. The step is therefore 0. On the contrary, when the wheels are oscillating 180 degrees out of phase ($\Delta\phi = 180$), there is no translation but pure rotation: the robot points left and right alternatively. When the phase difference is between 0 and 180, the movement is a combination of movements: turning left, going backwards turning right and going forward. Figure 6 shows the variation of the step with the phase difference. The step is maximum when the phase difference is 90 degrees, and does not depend on the amplitudes.

The amplitude (A) is directly related to the step size. As can be seen in the figure 7, for a given phase difference, when the amplitude is increased, the step is bigger.

The period (T) determines the time it takes for the robot to complete a step, but does not affect the trajectory.

B. Trajectory and direction

In the trajectory previously shown in figure 5, the robot is moving in the x axis direction ($\gamma = 0$). Initially, the two wheel angles are 0 ($\varphi_1 = \varphi_2 = 0$). In order to change the direction of the movement, the initial orientation θ should be change by means of the offsets (O) applied to the wheels (eq. 5). The robot will move in the same fashion, but in a new direction given by the following expression:

$$\gamma = \frac{r}{l} O \tag{8}$$

Therefore, the direction is controlled by means of the offset parameter. In figure 8, three trajectories in different directions are shown. The robot moves along the following directions: $\gamma = 28.3$ degrees ($O = 40$ degrees), $\gamma = 0$ ($O = 40$) and $\gamma = -28.3$ ($O = -40$). The trajectories are the same, but rotated γ degrees around the z axis.

Limited wheels impose a constraint on the trajectory direction. The robot gross translation movement direction is limited within the range of $[-\gamma_{max}, \gamma_{max}]$, as shown below. As the limited wheels angles φ_1 and φ_2 are restricted to the range $[-\varphi_{max}, \varphi_{max}]$, the following restriction is always satisfied:

$$|\varphi_i| \leq A + |O| \leq \varphi_{max}, \ \ i \in \{1, 2\} \tag{9}$$

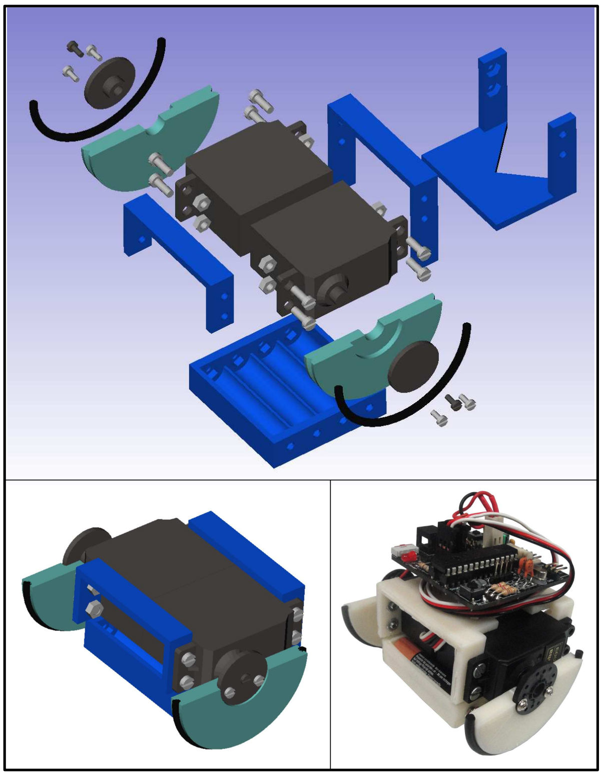

Fig. 10. The mobile robot built for the experiments. It is a modified version of the Miniskybot robot with limited wheels

In order to change the direction, $|O|$ should be different from zero. Due to the restriction 9, the amplitude should be less than $\varphi_{max} - |O|$. Applying the equation 8 the range for the amplitude is calculated:

$$A \in \left[0, \varphi_{max} - \frac{l}{r}|\gamma|\right] \tag{10}$$

It can be seen how the direction γ restricts the amplitude and consequently the step. When the offset is equal to φ_{max}, the amplitude is 0 and therefore there is no movement in that direction. The value of γ_{max} is then:

$$\gamma_{max} = \frac{r}{l}|\varphi_{max}| \tag{11}$$

In figure 9 is shown the relation between the step and the direction when $\varphi_{max} = 90$ and the oscillators have the maximum amplitude. The robot only can move in the directions less than 63.3 degrees. When $|\gamma|$ is decreased, the step is bigger (as the amplitude is bigger). The maximum step is obtained when the robot is moving along the x axis ($\gamma = 0$).

V. Experiments

A. Mobile robot with limited wheels

A mobile robot with limited wheels has been built for performing the experiments, shown in figure 10. It is a modified version of the Open Source Miniskybot [15], with a size of 71x87x62mm^3 and 200 gr in weight (including batteries). The chassis consist of 4 parts (in blue color in the figure): the front, the rear, the battery holder and the battery compartment. They have been printed in ABS plastic (Acrylonitrile Butadiene Styrene) using an Open Source Reprap-like 3D printer. The limited wheels are driven by two hobby servos, with a rotation range of 180 degrees ($\varphi_{max} = 90$). The wheels have also been built with the 3D-printer and are screwed directly to the servo horn.

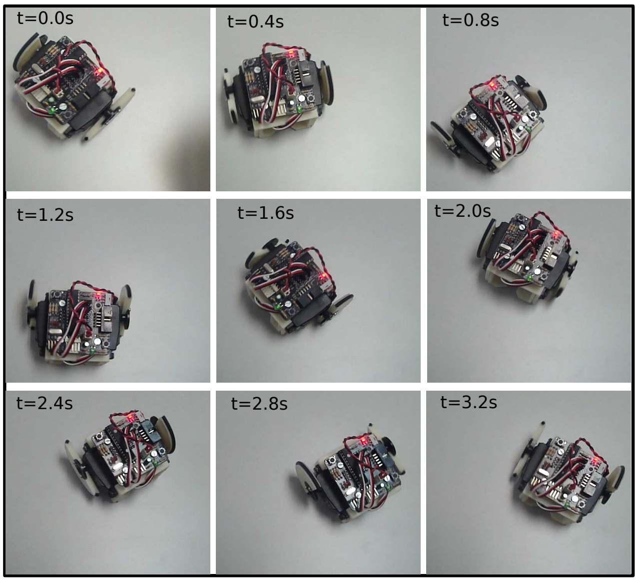

Fig. 11. Locomotion of the robot with limited wheels during two oscillation cycles

B. Locomotion with limited wheels

Several experiments on the locomotion of a robot with limited wheels have been conducted. They confirm the principle and demonstrate the viability of these new locomotion gaits for mobile robots. In figure 11 the locomotion of the robot is shown for two oscillation cycles (see the video attached to the paper or watch it on-line[1]). The robot moves with maximum step ($A = 90$, $\Delta\phi = 90$) along the x axis ($\gamma = 0$). The oscillation frequency has been set to 1/1.8=0.55Hz. The robot is initially at the rest position ($t < 0$, not shown in the pictures), with the wheels perpendicular to the x axis and the robot front pointing in the positive y axis (there is a red led on the robot front). When the robot is switched on ($t = 0$), the wheels are rotated to their initial positions given by the oscillators, $\varphi_1 = 0$, $\varphi_2 = 90$ and the robot turns to the right. From this position, the two wheels start oscillating and the robot moves sideways to the right.

The coordination of the two wheels is given by the phase difference. The "magic" value of 90 degrees combines the four movements equally and smoothly: turn left, go backwards, turn right and go forward generating that particular trajectory. During the periods of time in which the two wheels move in the same direction, the robot goes forward or backward. When they do in opposite directions, the robot turns left or right. The transitions among these

[1] http://goo.gl/2qv6V

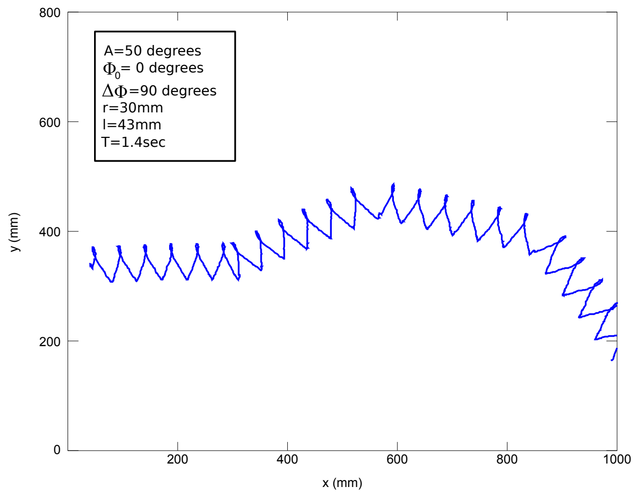

Fig. 12. A trajectory performed by the real robot. The robot was tracked and the trajectory recorded on a file

movements are smooth because they are propagated through the continuous generated oscillations.

C. Robot tracking and trajectory recording

In order to compare the real robot trajectory with those predicted by the kinematics, experiments on tracking the robot positions and recording the trajectory have been conducted. An infrared emitter has been located on the top of the robot, aligned with the center of mass. A Nintendo Wiimote controller has been situated 1.5 m above the robot, pointing towards the ground. This device has an infrared camera that is able to track the x and y coordinates of an infrared source and send them to a PC via bluetooth.

The software running on the PC of the setup receives IR positions and records the trajectory. Therefore the trajectory of the robot center of mass is obtained and stored into a file.

In figure 12 a trajectory of the real robot is shown. The robot was programmed to change the direction every five cycles. The locomotion parameters used were: $A = 50$, $\Delta\phi = 50$ and $\phi_0 = 0$. This experiment demonstrates the viability of controlling the robot direction.

VI. Conclusion and Future Work

All the mobile wheeled robots found in literature rely on the *rolling principle*: it is assumed that the wheels can rotate indefinitely. However, this principle does not work in the case of mobile robots with limited wheels. For coping with this, we propose the *swing principle*. Even though the available joint angle range of limited wheels is constrained, oscillatory movements are permitted. A correct coordination of these oscillations propels the robot sideways, performing a new locomotion gait not previously implemented by other researchers (to the best of our knowledge).

Motion can be controlled with only three parameters: the amplitude that determines the step size, frequency for setting the speed, and the offset for changing the direction of advance. A phase difference of 90 degrees guarantees a correct coordination of the wheels and maximizes the step.

The authors propose the use of other bio-inspired controllers such as CPGs (central pattern generators) or neural networks for controlling wheeled robots by means of the swing principle as future lines of research. Additional advances can be provided by research on new actuators to move limited wheeled mobile robots, such as artificial muscles (SMA) or other kind of linear actuators. Finally, the authors propose to implement this new motion principle on standard wheeled mobile robots for increasing their maneuverability, addressing situations in which normal functioning is impossible or developing applications such as 4-wheeled robot parking.

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